DEVELOPMENT OF HYDRODYNAMICS AT THE SIBERIAN BRANCH, ACADEMY OF SCIENCES OF THE USSR

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The classes of hydromechanics (mechanics of liquids and gases) problems studied by Siberian scholars has been defined to a significant degree by the interest of the founder of the Siberian Branch, Academician M. A. Lavrent'ev. His scholarly legacy includes studies on surface wave theory, the hydrodynamics of jet and detached flows, ground water motion beneath hydrotechnical installations, displacement of solid deformable bodies under the action of external forces, and hydrodynamic cumulation theory. A professional mathematician, Lavrent'ev supported studies to develop new mathematical methods for analytical and numerical simulation and analysis of natural processes. As a long-term student of nature, he proposed, and encouraged others to propose, experiments which might clarify the basic principles of yet incompletely understood phenomena. He presented many ideas and propositions for new formulations of hydromechanics problems, for example, on modeling of an explosion-driven mass of soil as an ideal incompressible liquid, on study of turbulent ring structures, explanation of the "Novorossisk Bora" (cold northern wind), solution of seeming paradoxes in asymmetric flow over a sphere, a new mechanism of wind wave formation, description of liquid flow in a bottom trench, problems involving tsunamis, etc. A description of original formulations in various fields of hydrodynamics with a detailed analysis of the physical factors controlling these phenomena can be found in his monograph (written in collaboration with B. V. Shabat) [1], which appeared in 1973.

Thanks to the initiatives and efforts of Lavrent'ev, his companions, successors, and hydromechanics students of the Siberian branch, hydromechanics blossomed. Its achievements have been recognized internationally and are an object of pride for the Siberian scholars. The present study is dedicated to a brief overview of these achievements.

Group Analysis of Differential Equations. The most widely used mathematical model for liquid and gas motion involves differential equations with supplemental conditions. An important stage in study of such models is qualitative analysis, consisting of clarification of correctness of the model and a search for sufficiently broad classes of particular solutions. The latter is usually a nontrivial task in view of the nonlinearity of the hydromechanics equations.

In solving the problem of seeking particular solutions, the concept of using special symmetry properties of the differential equations has proved fruitful. This special symmetry involves the fact that the equations permit a continuous group of transformations of the set of independent and dependent variables. The concept of a permissible group had already been introduced in the studies of the Norwegian Sophus Lie at the end of the 19th century. But after Lie's works his theory was not widely employed in hydromechanics studies.

Beginning in 1958, L. V. Ovsyannikov, followed by his students, expended much effort on the "reanimation" of Lie's inheritance in the direction of applications of the theory of continuous group transformations to the problems of mathematical physics in general, including hydromechanics. This field of study revealed the appelation "differential equation group analysis." The major achievements in this field of study were presented in monographs by Ovsyannikov [2] and Ibragimov [3].

Algorithms were developed in detail for group classification of differential equations and construction of classes of invariant and partially invariant solutions. Group classifications were performed for the equations of nonlinear thermal conductivity, those of an ideal incompressible liquid, the gas-dynamics equations, the Navier–Stokes equations, boundary-layer equations, those of transsonic gas flow, and other systems of hydromechanics equations.

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For the equations of motion of a gas with equation of state $p = A(S)\rho^\gamma$ in dimensionality n (with values n = 1, 2, 3 having physical meaning), where $x \in R^n$ and $\mathbf{u} \in R^n$,

$$\rho_t + \operatorname{div} \rho\mathbf{u} = 0, \quad \mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} + \rho^{-1}\nabla p = 0, \quad S_t + \mathbf{u}\cdot\nabla S = 0 \tag{1}$$

values of the adiabatic index $\gamma = (n + 2)/n$ were found, leading to an expansion of the permissible group [2] and new conservation laws [3] of the form $\tau_t + \operatorname{div}(\tau u + \xi) = 0$, where

$$\tau = t(\rho|\mathbf{u}|^2 + np) - \rho\mathbf{x}\cdot\mathbf{u}; \quad \xi = p(2t\mathbf{u} - \mathbf{x}).$$

It was established that solutions of the dual wave type are isentropic for system (1) in the case n = 2. It was shown for Chaplygin's gas-dynamic equation $\psi_{\sigma\sigma} + K(\sigma)\psi_{\theta\theta} = 0$ that all its successful approximations which permit an efficient solution of the boundary problems are caused by the maximal width of the admissable group [4].

The problem of seeking particular solutions of differential equations then received a rational group basis. The group analysis methods also proved effective for study of hydromechanics boundary problems. Menshikov [5] studied questions of invariance of solutions of the Cauchy problem, characteristics, and strong discontinuities, in particular, invariant extension of the invariant solution of system (1) through a shock wave. Studies of the invariance of problems with a free boundary for the Navier–Stokes equation, performed by Pukhnachev [6], made it possible to find a solution to a wide range of problems of this type.

Further development of Lie's theory in [3], involving development of the Lie–Backlund group apparatus, led to construction of nontrivial conservation laws for a number of nonlinear evolutionary equations, describing wave processes in liquids and gases. The many other results obtained by differential equation group analysis in mathematical physics can be found in [2, 3] and the journals cited therein.

Motion with a Free Boundary. This term distinguishes a class of liquid and gas motions in which a portion of the moving mass boundary is "free", i.e., not a solid impermeable wall, but is determined by some other law of contact interaction with the surrounding medium. Typical examples are action on the free boundary of distributed pressure or contact of the mass under study with another liquid. Motion with a free boundary is widespread in nature, including jet and cavitation flows, wave motion on the surface and in the depths of a stratified ocean, free motions of a finite liquid mass when thrown, etc. In the corresponding hydrodynamics problems a new unknown appears — the region of definition of the solution (in fact, the boundary thereof).

Steady-state (settled) and nonsteady-state (unsettled) motions of liquid with a free boundary differ significantly. The former have a large history of many fruitful studies with precise formulation in the works of Euler, Kirchhof, Chaplygin, Nekrasov, Lavrent'ev, and other eminent scholars. At the same time, as regards nonsteady-state motions of this type, in fact no precisely formulated results were ever presented before those of Siberian scholars. Therefore development of the corresponding precise theory of nonsteady-state motions of a liquid with a free boundary can be considered one of the major attainments of Siberian science.

We consider here potential motions of an ideal incompressible liquid with a free boundary Γ_t, limited by an unknown region $\Omega_t \in R^3(x)$ (where the subscript t denotes time dependence), located within the field of mass forces with a potential G and constant pressure on Γ_t. The velocity potential $\Phi(t, x)$ must be a harmonic function within Ω_t and at $x = X \in \Gamma_t$ must satisfy the kinematic and dynamic conditions

$$\mathbf{X}_t = \nabla\Phi, \quad 2\Phi_t + |\nabla\Phi|^2 = 2G. \tag{2}$$

To Eq. (2) we add the initial conditions

$$\mathbf{X}|_{t=0} = \mathbf{X}_0, \quad \Phi|_{t=0} = \Phi_0, \tag{3}$$

where the function Φ_0 is harmonic in the known region Ω_0 with boundary Γ_0 upon which $x = X_0$. The vector conditions of Eqs. (2), (3) in fact reduce to scalars, since Γ_t can be specified with one equation in $R^3(x)$.

The difficulties in analysis of the problem of Eqs. (2), (3) are caused by its obvious nonlinearity, as well as its nonlocalness. The latter follows from the dependence of the value of $\nabla\Phi$ at each point of Γ_t on all values of the potential Φ itself on Γ_t. Thus the problem of Eqs. (2), (3) does not simplify in any conventional space due to the continuous loss of smoothness in the solution with increase in time t. A new mathematical apparatus is needed to overcome these difficulties.

Fundamental in this regard is Oxyannikov's study [7], which first revealed the possibility solution of linear nonlocal Cauchy problems based on the new concept of a singular operator in the sale of Banach spaces. Osyvannikov also showed [8] on the basis of model formulations that a problem with a free boundary could be correctly formulated within the class of analytical functions. This study [8] also obtained an apriori evaluation for small perturbations of an arbitrary solution of Eqs. (2), (3), and proved the uniqueness of the solution of the corresponding linearized problem.

The next step was made by Nalimov [9], who first proved the solubility (for small t) of the nonlinear Cauchy—Poisson problem of waves in water, in which $G = -gy$, while Ω_t is defined by an impermeable wall y = 0 (the bottom) and a free boundary $y = h(t, x) > 0$ (the planar problem). The existence of a solution within the class of analytical functions was established by refinement of Schauder estimates of arbitrary order on the boundary of the region and use of the results of [7].

Justification of Approximations in Wave Theory. The technique of analysis of problems with a free boundary within the class of analytical functions was perfected by Ovsyannikov [10] on the basis of the concept of the quasidifferential operator within the scale of Banach spaces. This permitted a significant simplification of the proof of solubility of such problems and production of a precise justification of the approximations used in the theory of liquid wave motions, such as the linear, "shallow water," etc.

The concept of precise justification refers to problems in the formulation of which there appears (or is introduced) the parameter ε, with the approximate formulation being obtained by the formal transition $\varepsilon \to 0$. In this situation the solution of the original problem is, let us say, $u(\varepsilon)$ and it is necessary to evaluate the deviation thereof from the solution $u(0)$ in the approximate formulation. For example, for justification of the linear theory, where the solutions is represented in the form $u(\varepsilon) = u_0 + \varepsilon u_1(\varepsilon)$, where u_0 is the fundamental exact solution, independent of ε, and $\varepsilon u_1(0)$ is its linear approximation, it is necessary to prove that $u_1(\varepsilon) - u_1(0)|| \to 0$ as $\varepsilon \to 0$. In the "shallow water" theory, which produces an approximation widely used in the Cauchy—Poisson problem, the unknowns are h and φ (liquid depth and value of the velocity potential on the free boundary). When the solution is represented in the form $h(\varepsilon) = \varepsilon h_1(\varepsilon)$, $\varphi(\varepsilon) = \varepsilon^{1/2}\varphi_1(\varepsilon)$, the functions $h_1(0)$, $\varphi_1(0)$ satisfy the "shallow water" equations. Here we must evaluate the dependence upon ε

$$\delta = ||h_1(\varepsilon) - h_1(0)|| + ||\nabla\varphi_1(\varepsilon) - \nabla\varphi_1(0)|| \tag{4}$$

given the condition that the initial data in the Cauchy—Poisson problems for $(h_1(\varepsilon), \varphi_1(\varepsilon))$ and $(h_1(0), \varphi_1(0))$ are identical.

A precise justification of the "shallow water" theory was first offered for periodic waves [11], and then for "finite" waves in the planar problem. The results were obtained within the scale of Banach spaces of analytic functions where, for example, the norm of the function u(x) is defined by its Fourier transform $\hat{u}(\xi)$

$$\|u\|_\rho = \int_{-\infty}^{+\infty} e^{\rho|\xi|}\,|\hat{u}(\xi)|\,d\xi \quad (\rho > 0). \tag{5}$$

Analogous results were obtained by Makarenko [12] for problems involving motion of a two-layer liquid, and for three-dimensional waves. In all these cases, using a norm in the form of Eq. (5) the value of Eq. (4) was found to be $\delta < C\varepsilon$ (with the constant C being independent of ε).

A new approach to the theory of Cauchy—Poisson waves in the planar problem was developed by Nalimov [13]. Using the technique which he developed for precise evaluation of nonlinear pseudodifferential operators, he proved the existence and uniqueness of the solution of the Cauchy—Poisson problem within the class of functions of finite smoothness and within the Jervet class, and gave an exact justification of linear theory. Further development of this technique led to justification of the "shallow water" theory within the class of finite smoothness functions with the somewhat weaker estimate $\delta < C\varepsilon^{1/2}$ of the deviation of Eq. (4).

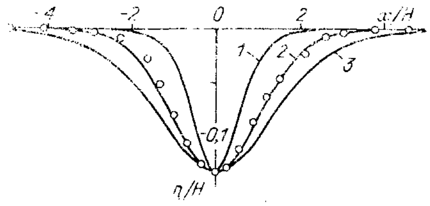

Fig. 1

The overall outcome of studies performed on justification of approximate theories of waves involve not only providing such approximations with a reliable theoretical basis, but also a newly developed analytical apparatus, which can be used for study of analogous problems. The results mentioned in this section are presented in detail in [14].

The second approximation theory developed in [14], which produces simple finite expressions for the basic parameters of combined waves and smooth winds in a two-layer liquid beneath a lid was subjected to experimental verification of Bukreev and Gavrilov [15, 16]. Figure 1 shows a comparison of soliton profiles calculated by this theory (line 2) with experimental data (circles), obtained at a relative depth of the lower layer of 0.737 and density ratio of 0.8. For these conditions the theoretical value of wave propagation velocity is 15.1 cm/sec, while the experimental value is equal to 14.9 ± 0.3 cm/sec. Figure 1 also shows soliton profiles calculated for the same initial data by the theory of Kakutani and Jamasaki [J. Phys. Soc. Jpn., 45, No. 2 (1978)], using the Korteweg–de Vries equations (line 1, wave speed 16.1 cm/sec), and by the theory of Keulegan [J. Res. Nat. Bur. Stands., 51, No. 3 (1953)] (line 3, wave speed 15.4 cm/sec).

Steady State Flows with Free Boundaries. Powerful new achievements have also been attained in this broad field of hydromechanics, already enriched by classical results. In the review [17], coauthored by Lavrent'ev, four problems which the authors considered "fundamental to wave theory" were noted. At the present time significant progress has been achieved in all four of these problems. One of these, the creation of a theory of unsettled waves, has already been discussed above.

The second problem is related to the absence of an exact theory of spatial flows with a free boundary. The first major step here was made by Plotnikov [18]: in precise formulation the existence of a three-dimensional dual-period wave system on the surface of a finite depth liquid flow was proven. Solutions of the corresponding linear problem, proportional to $\exp i(\omega nx + my)$

$$\omega^2 n^2 = \lambda k \operatorname{th}(h_0 k),\ k = \sqrt{\omega^2 n^2 + m^2}, \tag{6}$$

where n, m are integers, h_0 is the depth of the unperturbed flow with velocity u_0 in the direction of the x-axis, ω is the ratio of the periods along the y- and x-axes, $\lambda = h_0^{-1}\,\mathrm{Fr}^{-2}$ with Froud number $\mathrm{Fr} = u_0/\sqrt{gh_0}$ (force of gravity acting along the z-axis).

The major difficulty in solution of the spatially nonlinear problem is caused by its principal difference from the planar problem: the set of λ values obtained from Eq. (6) for all possible integer pairs (n, m) is full everywhere in the positive semiplane (continuous spectrum). Nevertheless Plotnikov proved the existence of a single-parameter family of solutions of the nonlinear problem (with parameter ε), in which the liquid depth is represented by

$$h(\varepsilon, x, y) = h_0 + \varepsilon \cos \omega nx \cdot \cos my + \varepsilon\eta(\varepsilon, x, y)$$

with $\|\eta\| < C\varepsilon$. The result was obtained by using a variant of the Nash–Moser formula, producing a special iteration algorithm in the form of a combination of Newton's method with smoothing of successive approximations in the scale of Banach spaces of dual-period functions of finite smoothness.

The third problem involves planar waves of maximum (limiting) amplitude, for which Stokes derived a slope of 30° at the peak of the waves. Since this was not done precisely, since then one has spoken of the Stokes hypothesis. Although the existence of maximal amplitude waves was established by the English mathematician Toland in 1978, the Stokes hypotheses remained an open question. Using a precise analysis of the extension of the analytical solution through the free boundary Plotnikov [19] refined Toland's result and finally the validity of the Stokes hypothesis.

Finally, the fourth problem concerned the principal difficulties of analysis of flows over an uneven bottom at Froud numbers less than unity. One planar problem of this class was solved by Nalimov [20]. He studied bifurcation of a liquid flow with a velocity u_0 and depth h_0 above a bottom with an equation $y = \varepsilon h(x)$ with a sufficiently smooth function $h(x)$ decaying rapidly as $|x| \to \infty$ and small parameter ε with the assumption that the Froud number $u_0/\sqrt{gh_0} < 1$. It was proven that a solution exists, which as $x \to -\infty$ exists to a uniform flow moving at velocity u_0, while for $x \to +\infty$ the flow behaves like a periodic wave with period $2\pi/\omega$, determined by linear theory. The principal point of Nalimov's theory is the unique expansion of the unknown function u(x) into two components

$$u = u^0 + \theta u^+, \tag{7}$$

where $u^0(x)$ decays exponentially as $|x| \to \infty$, $u^+(x)$ is a $2\pi/\omega$-periodic function, and the standard factor $\theta(x) \in C_\infty$ is equal to zero at $x < 0$ and unity at $x > 1$. The proof involves a special normalization of functions in the form of Eq. (7) within Banach spaces of finite smoothness functions and the technique of pseudodifferential operators.

A supplementary condition in this result is of interest: the Fourier transform of the derivative $h'(x)$ must satisfy the inequality $\hat{h}'(\omega) \neq 0$. The question of whether this condition has some physical meaning, and how generally necessary it is, is still open.

Steady-state gas-dynamic problems provided Lavrent'ev with a model for creation of the theory of quasiconformal mapping [21]. Planar ($k = 0$) and axisymmetric ($k = 1$) potential gas flows are described by a nonlinear system of equations for the velocity potential $\varphi(x, y)$ and the flow function $\psi(x, y)$:

$$y^k \rho \varphi_x = \psi_y, \; y^k \rho \varphi_y = -\psi_x. \tag{8}$$

Given modulus of the velocity $q = |\nabla\varphi|$ and density $\rho = \rho(q)$ of the type of system (8) the sign of the value $d(q\rho)/dq = \rho c^{-2}(q^2 - c^2)$ is defined, where c is the speed of sound: elliptical for $q < c$ and hyperbolic for $q > c$. The wide class of boundary systems for system (8) is related to conditions on the boundary Γ, consisting of specified impermeable walls Γ_1 and free boundaries Γ_2 (Fig. 2):

$$\partial\varphi/\partial n|_\Gamma = 0, \quad \varphi_y/\varphi_x|_{\Gamma_1} = \operatorname{tg}\theta(x), \quad q|_{\Gamma_2} = q(x, y). \tag{9}$$

Lavrent'ev's main idea was that solutions of the elliptical system (8) could be accomplished by quasiconformal mapping of the plane (x, y) into the plane (φ, ψ). He solved the planar problem of infrasonic potential flow of a gas in a channel with curvilinear walls.

Using developments of the theory of quasiconformal mapping Monakhov [22] proved the solubility of a number of planar strictly elliptical problems of the form of Eqs. (8), (9). Generalizations of these results to doubly-bound regions obtained by S. N. Antontsev, and to turbulent flows of an incompressible liquid by P. I. Plontnikov, were also presented in [22]. The study of axisymmetric flows of this type was begin by Plotnikov [23]. For gas flows Antontsev [24] first proved that for strictly sonic flow on the free boundary an axisymmetric jet equalizes itself at a finite distance.

Linear Waves. Linear wave theory is based on the methods of Fourier analysis and dispersion relationships. The latter can be found in explicit form [see, e.g., Eq. (6)] only in the simplest cases. Any inhomogeneities, including initial stratification of unevenness of the bottom, lead to complex spectral problems. The difficulties which then develop are especially characteristic of the most interesting and practically pressing problems in spatial wave propagation. Therefore within linear theory wide use has been made of various asymptotic approximations, including steady phase, long and short wave asymptotes, etc. On the whole linear wave theory has become a broad field in which hundreds of studies are published every year. Beginning in 1972, the M. A. Lavrent'ev Hydrodynamics Institute of the Siberian Branch, Academy of Sciences of the USSR has published annotated bibliographic indices [25-21], which reflect the contribution to this field of Siberian scholars. Two particular results will be discussed below.

The waveguide problem, formulated by Lavrent'ev in connection with tsunamis, was solved by Garipov [28] (the same question was studied in approximate formulation in [29]). A linear variant of the Cauchy–Poisson problem of Eqs. (2), (3) was considered for waves propagating in an infinite layer of homogeneous liquid $-1 + \varepsilon h(x) < z < 0$ with function $h(x) \geq 0$ nonzero only over a finite interval, and small parameter $\varepsilon > 0$. Here the geometric form of the bottom

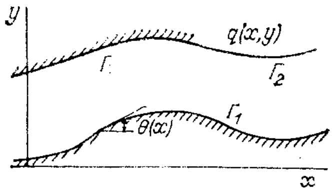

Fig. 2

$z = -1 + \varepsilon h(x)$ is a submerged mountain ridge — a cylindrical elevation along the y-axis. For a velocity potential Φ with values $\varphi = \Phi$ on the free surface $z = 0$ we obtain a Cauchy problem

$$\varphi_{tt} + K\varphi = 0,\ \varphi|_{t=0} = f_0,\ \varphi_t|_{t=0} = f_1,$$

where the operator $K\varphi = \Phi_z|_{z=0}$ contains within itself all the information on the geometry of the region. The form of the wave is determined by the equation $\zeta = -\varphi_t$. In the representation $K = K_0 + \varepsilon k$, the operator K_0 corresponds to a planar bottom $z = -1$, while the small increment εk depends on the form of the bottom, i.e., on the function $h(x)$ and the parameter ε.

A Laplace transform with respect to t leads to study of the resolvent of the operator K, which has both a discrete and a continuous spectrum. This problem was studied most fully by Garipov in [30], which considered the spectral representation $\theta(\sigma, \nu) = \sqrt{\sigma^2 + \nu^2}\ \mathrm{th}\ \sqrt{\sigma^2 + \nu^2}$ of the operator K_0, which as an analytical mapping of the complex plane $\sigma \to \theta$ (for fixed ν) is two-layered. Therefore the resolvent of the operator K_0 extends analytically into a two-layered Riemann surface with branching point $\theta_0 = \nu\ \mathrm{th}\ \nu > 0$. It develops that if the area $S = \int h(x)dx$ is positive, then in the first layer of the Riemann surface there exists an isolated eigenvalue λ_1 of the operator K, such that $0 < \lambda_1 < \theta_0$. The corresponding solution attenuates with propagation along the ridge by a law $t^{-\gamma}$, where γ is determined by the value of S and takes on one of the values 1/2, 1/3, 1/4 in contrast to a planar bottom, where the attenuation law is t^{-1}. Thus it was shown that an underwater ridge can act as a waveguide for surface waves.

Sturova's results [14] involve study of surface and internal waves, developing in a density-stratified liquid with various excitation methods. A comparison of the asymptotic patterns of waves produced by uniform horizontal motion of a body and by compression of the mixing zone [31] revealed that (in the presence of a density discontinuity) in the second case internal waves produce practically no distortion of the boundary and do protude above the density discontinuity. It was also established that upon imposition on the translational motion of a body of additional oscillations it is possible to excite waves ahead of the body with an abrupt increase in wave amplitudes at certain body oscillation frequencies. The effect of decrease in wave resistance of a longitudinally oscillating body as compared to purely translational motion was observed.

Flow of a Viscous Liquid with Free Boundaries. Pukhnachev [32, 33] considered the cycle of problems involving free boundaries for the Navier-Stokes equations in precise formulation. Consideration of surface tension σ introduces into the equations together with the Reynolds number $Re = V\ell/\nu$, a second similarity criterion, the Weber number $We = \rho V^2 \ell/\sigma$, the values of which defines bifurcation flow regimes. Solutions of problems of the motion of a liquid film coating the surface of a horizontally rotating cylinder in a gravity field and of equilibrium forms of a weightless capillary liquid partially filling a cylindrical capillary and rotating together with the cylinder about the latter's axis at constant velocity were achieved [34].

Approximate models of wave formation in adherent films of a viscous liquid were presented in detail in a monograph by Nakoryakov et al. [35]. This problem was first solved in exact formulation by Pukhnachev [36], who proved the existence of rolling waves as a bifurcation motion of the travelling wave type against a background of a planar flow with rectilinear trajectories and semiparabolic velocity profile. A single-parameter family of solutions exists for any Weber number and sufficiently small Reynolds number.

Nonsteady state motions of a ring of viscous incompressible liquid with free boundaries were considered by Bytev [37]. It developed that in the absence of surface tension the internal radius of the ring increases without limit, depending on the initial peripheral velocity distribution, either linearly with time t (like a ring of ideal liquid) or by a $t^{1/2}$ law. When surface tension is considered, a rotating ring behaves completely differently. Here the characteristic parameter β (the Weber number) is defined by the initial conditions in terms of the angular momentum and the ring area. Lavrent'eva [38] proved the existence of a critical value $\beta_* \cong 5.17$, dividing different regimes of steady state motion: for $\beta > \beta_*$ there are two steady state solutions, while at $\beta = \beta_*$ there is one, and for $\beta < \beta_*$ such solutions do not exist. For unsteady state motions with $\beta \geq \beta_*$ or as $t \to \infty$ the solution tends to steady state, or after a finite time the ring transforms to a circle. The existence and uniqueness of the corresponding solution of the Navier-Stokes equations are proved. Thus it was established that for the given description of the process the transformation of the ring into a circle is irreversible.

Phenomena within the boundary layer of a nonuniformly heated liquid are related to the temperature dependence of surface tension. The intensity of such boundary layer flows is determined by the Marangoni number $Ma = \ell\sigma_T\Delta T/\rho\nu^2$, where σ_T is the surface tension temperature coefficient, and ΔT is the characteristic temperature differential. Asymptotic simplification of the problem as $Ma \to \infty$ leads to a nonclassical boundary problem for the system of Prandtl equations: instead of the condition of adhesion at a rigid surface the tangent stress on the free boundary is specified. The first results for the two-dimensional nonsteady state problem of equations describing Marangoni boundary layers were achieved by Pukhnachev [39]. Using the theory developed by Kuznetsov [40] an asymtotic method was developed for calculation of thermocapillary convection in a liquid column, the lateral surface of which is free, while constant (but different) temperature values are specified on the face surfaces.

A contribution to the theory of the Prandtl boundary layer was made by Khusnutdinova [41], who indicated sufficient conditions for existence "on the whole" of a two-dimensional steady state boundary layer with increase in pressure down the flow.

Inertial Motion of a Finite Liquid Mass. The development of Lavrent'ev's idea [42] of modeling motion of an explosion-driven soil mass as motion of an ideal incompressible liquid led to the question of stability of the motion of a finite liquid mass with free boundary. This question has meaning if an exact solution of the Euler equations defined for the time interval $0 \leq t < \infty$ is known. Such solutions exist, for example, for the class of motions with velocity field linear in the coordinates [43]. In particular, they describe motion of liquid ellipsoids and lead to a Lagrangian dynamic system on a manifold [44]. A stability analysis performed by Pukhnachev and Andreev showed that ellipsoid motions were stable in integral norms, but unstable in a uniform metric due to the possibility of formation of long "whiskers," localized in narrow regions of the free boundary [45].

In the general case of potential motion of a liquid mass having volume V, density ρ, and kinetic energy K, the diameter of the liquid mass $d(t) \to \infty$ as $t \to \infty$, Nalimov and Pukhnachev [46] obtained the asymptotic estimate

$$d(t) \geqslant 2(K/\rho V)^{1/2}(t + O(1)).$$

An analogous result is also valid for turbulent motions, if the measure of the vorticity $|\mathrm{curl}\,v|\,|D(v):D(v)|^{-1/2}$ with deformation rate tensor D(v) for the velocity field v is uniformly limited by unity. Highly turbulent motions of a liquid volume may be compact for all t. Pukhnachev [47] found a family of rotationally symmetric steady state motions of a finite liquid mass with a toroidal free surface — a hydrodynamic analog to closed plasma configurations.

Spatial shock-wave interactions. Generalization of classical results on propagation and interaction of strong discontinuities with one-dimensional gas motions in spatial motions involves the necessity of considering the possible curvilinear form of the front. Studies in this field were begun by Teshukov [48], who was the first to establish the existence of piecewise-analytical solutions for all possible configurations which develop upon decay of an arbitrary discontinuity (of the first sort) on a specified analytical surface. He also solved the general problem of regular interaction of two curvilinear shock waves propagating in a space $R^3(x)$ [49].

Let Γ_t be the line of interaction of the incident fronts of the shock waves at time t (Fig. 3), v_{n1} and v_{n2} being the normal propagation velocities of the fronts relative to the gas. Then the relative normal velocity w of propagation of the curve Γ_t is given by the expression

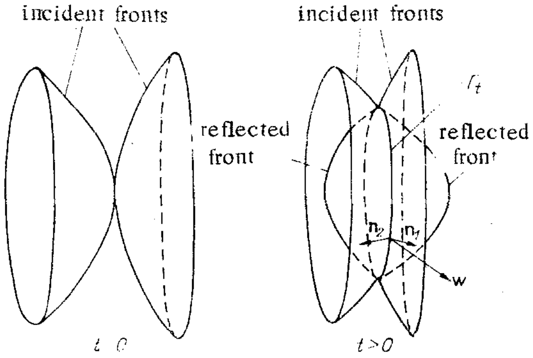

Fig. 3

$$w = \frac{n_2 - n_1(n_1 \cdot n_2)}{|n_1 \times n_2|^2} c_{n1} + \frac{n_1 - n_2(n_2 \cdot n_1)}{|n_1 \times n_2|^2} c_{n2},$$

where n_1 and n_2 are unit vectors of the normals to the incident fronts. At the initial moment t = 0 of first contact of the fronts $|w| = \infty$. The existence of a piecewise-analytical solution of the problem has been proven for the time interval defined by the inequality $|w| > c$, the speed of sound in the reflected fronts. For t > 0 the regular interaction stage is always described uniquely by a "weak" reflected discontinuity in contrast to steady state oblique reflection, where the question arises of choosing one of the possible discontinuities – "weak" or "strong."

The principles of shock wave propagation through a specified background (state ahead of the wave) were studied by Blokhin [50]. For a natural formulation of the initial-boundary problem the existence of a solution within the class of finite smoothness functions has been proved, and estimates have been obtained from which the stability of the shock wave follows.

Hydraulic Processes. This general designation combines studies on mathematical modeling and calculation of unsteady state liquid motions in open waterways, channels, and tubes. Efforts in this pressing practical direction were organized by O. F. Vasil'ev. On the basis of the Saint-Venant and Boussinesq models, using the methods of Khristianovich, Arkhangel'skii, and Godunov, a number of authors developed effective algorithms and application program packages for hydraulic calculations of floods and large rivers, motions of a continuous wave upon destruction of a dam, flows in complex waterways with rings and branchings, and hydraulic shock phenomena in pipe systems. A qualitative description of results obtained in this field was given in [51, 52], which also contains a complete listing of published studies.

A principal contributions to this field was creation of calculation methods for branched one-dimensional hydraulic systems. Here a new type of boundary problem arises for the system of differential equations with partial derivatives specified on complexes (graphs). The problem consists of considering the topological structure of the complex in formulating consistent boundary conditions at the peaks of adjacent edges, insuring correct formulation of the problem on the complex as a whole. Atavin produced a solution [53] for subcritical flows in river waterway systems, forming a "tree" type complex. A monograph by Voevodin and Shugrin [54] described solution of this problem for a number of concrete cases, for example, hydroshock phenomena, unsteady state gas motions in a tube system, etc. Even at present this problem in its general form is far from complete solution.

Filtration Flows. Studies of filtration flows were commenced by P. Ya. Kochina. At her initiative efforts were exerted in the field of mathematical modeling of moisture and salt transport processes in soils during soil improvement. Also studied were phenomena such as formation of fresh water lenses above salt water, combined motion of soil and surface waters, unsteady-state moisture—salt transport in the aeration zone, drainage flows, salt accumulation in a root-containing soil layer, etc. Some achievements in this direction are presented in [55]. The next stage in development of filtration theory with consideration of complex hydrogeological conditions and other factors of practical importance, a multifaceted validation of the mathematical models used – from establishing the correctness of boundary problem formulations to cretion of programs and carrying out concrete calculations, was described by Antontsev et al. [56].

Questions of steady state filtration lead to problems with an elliptical system of differential equations. One of the practical problems is determination of filtration flows in a homogeneous soil layer, passing from a system of channels to interposed drains in the presence of infiltration or evaporation from the free surface. For the case of a planar periodic system of channels and drains Émikh [57] found a solution of the linear problem by the conformal mapping method. Analysis of nonlinear processes required methods more powerful than Lavrent'ev's conformal mapping theory. The development of this theory is presented by Monakhov [22] who, in particular, considered problems of nonlinear filtration and established the correctness of formulation of the basic nonlinear boundary problems.

Study of filtration of immiscible liquids is of practical importance in connection with the problem of increasing output of petroleum and gas formations. For this purpose a multifaceted analysis was performed of the well known Masket–Leverett model, consisting of continuity equations and Darcy's law for each component with consideration of capillary pressure on the interphase boundary. The first results in the linear variant of this model with a numerical realization were achieved by A. N. Konovalov [58]. A complete analysis of correctness of the fundamental boundary problems, which led to discovery of unusual effects, was performed by Monakhov and Antontsev [59].

The original model was reduced to a quasilinear system for "reduced" pressure $p(x, t)$ and saturation $s(x, t)$

$$ms_t = \operatorname{div}(K_0 a \nabla s + K_1 \nabla p + f_0), \quad \operatorname{div}(K \nabla p + f) = 0 \tag{10}$$

with $m(x) > 0$, symmetrically positively defined filtration tensor for the homogeneous liquid $K_0(x)$, phase permeability tensors $K_1 = k_{01}K_0$, $K = kK_0$, and functions $a(s)$, $k_{01}(s)$, $k(s)$, $f_0(x, s)$, $f(x, s)$ defined by the porosity of the medium. Boundary conditions for system (10) are specified on impermeable surfaces as well as the boundaries of the well and contacts with unmoving liquid. An initial value $s(x, 0)$ is specified for the saturation. In this formulation the specifics of two-phase filtration consist of the fact that saturation must satisfy the inequalities $0 \le s \le 1$, while for $s = 0$ (or $s = 1$) degeneration of system (10) occurs, where simultaneously $k_{10}(0) = 0$ and either $a(0) = 0$ or $a(0) = \infty$.

Proof was offered in [59] that the principle of the maximum was valid for the solutions, confirming satisfaction of the inequalities $0 \le s \le 1$, and with certain additional conditions, the inequalities $\delta \le s \le 1 - \delta$ with constant $\delta > 0$. The following solution properties were also established: for the degeneration $a(0) = 0$ the propagation speed for perturbations from the initial state proves finite, while for $a(0) = \infty$ after a finite time the solution stabilizes to $s = 0$ over the entire region of its definition. In other words, despite the overall diffusion character of the filtration process, in the cases indicated the pure phase region is filled by the mixture by propagation of some front at a finite velocity, or else after a finite time the entire region is purified of one of the phases.

Steady State Turbulent-Potential Flows. Observations have shown that near a rough body, a projection or identation in the bottom region appears in which liquid motion occurs over closed trajectories. In the 1960's Lavrent'ev proposed the following scheme of two-dimensional flows of this type within the framework of the ideal incompressible liquid model: in the circulation zone motion occurs with a constant vorticity, while outside it is potential with the velocity field being continuous everywhere. Studies of this topic are considered in detail in [1].

The nonlinear models with "pasting" of potential and turbulent flows which develop in this model were the subject of a number of studies [60-62], which clarified the fact that in individual cases it is possible to prove the existence of a solution. In addition numerical calculations performed demonstrated that the problem with "pasting" can have more than one solution – this question of uniqueness remains open at present.

Turbulent Rings. When a finite volume which is a part of an infinite viscous liquid is impulsively set in motion at constant velocity, after some time the motion takes on the form of a turbulent ring propagating within the liquid. Quantitative description of this phenomenon is one of the most complex and pressing problems of hydromechanics.

At the suggestion of S. A. Khristianovich, Onufriev [63] performed a theoretical study of the ascent into the atmosphere of the characteristic mushroom-like cloud produced by a powerful explosion, which is similar to a turbulent ring. The action of repulsive forces on the heated gas cloud, inhomogeneity of the atmosphere, and the turbulent character of the motion were considered. On the basis of conservation laws and a number of empirical relationships a model of this phenomenon was constructed in the form of a system of ordinary differential equations. The numerical solution determined the dimensions of the ascending cloud and the ascent height as functions of time with good agreement with experimental data.

Lavrent'ev called attention to two significant properties of turbulent rings: the relatively low resistance to motion and the ability to transport an impurity. At his initiative many experiments were performed for the purpose of determining the structure and parameters of turbulent ring motion. The experiments of Akhmetov and Kisarov [64] measured velocity fields in turbulent rings. A. A. Lugovtsov, B. A. Lugovtsov, and V. F. Tarasov [65] studied the laws of motion of turbulent rings formed in various manners over the range of initial radii 1-200 cm, velocity 0.1-100 m/sec, and Reynolds number 10^3-10^7.

Analysis of the experimental results led to the following conclusions. For Reynolds numbers Re > 10^4 the liquid motion in the ring is turbulent. Upon combination of the ring formation stage its radius R(t) increases linearly with distance traversed L(t):

$$R(t) = R_0 + \alpha L(t) \ (\alpha = \text{const}). \tag{11}$$

The quantity α can be assumed quite precisely in experiment and lies in the range 10^{-2}-10^{-3}. It depends on ring creation conditions and defines the structure of the average motion within the ring. Tarasov's experiments [66] showed that beginning at a Reynolds number of the order of 10^5, α arrives at a universal value of $3 \cdot 10^{-3}$.

On the basis of these experimental facts and the law of conservation of turbulent momentum $\mathbf{P} = \frac{1}{2}\int \mathbf{r} \times \boldsymbol{\Omega} dv$, where Ω is the average vorticity, Lugovtsov [67] developed the hypothesis of self-similarity of motion and structure of the turbulent ring, with the unique defining parameter being the turbulent momentum P_0. This hypothesis agrees with Eq. (11) and leads to a law of motion

$$L(t) = \frac{R_0}{\alpha}\left[\left(1 + \frac{4\alpha U_0}{R_0} t\right)^{1/4} - 1\right].$$

The hypothesis of self-similarity of turbulent ring structure leads to a time dependence of the turbulent viscosity coefficient

$$\nu_*(t) = \lambda P_0^{2/3} t^{-1/3} \quad (\lambda = \text{const})$$

and formulation of a boundary problem to find the vorticity distribution. In the system of differential equations of this problem λ plays the role of a small parameter in the lower order derivatives. In the limit of "vanishing viscosity" $\lambda \to 0$ Lugovtsov [68] formulated a boundary problem of the "pasting" type, containing no empirical constants, which was solved numerically.

Comparison of calculation results with experimental data on vorticity measurement indicated a shortcoming of this theory: in reality because of "gyroscopic elasticity" of the rotating liquid, within the core of the vortex, turbulence is suppressed. Empirical dependences were proposed for turbulent viscosity as a function of the analog of the Richardson number for the rotating liquid, which allowed achievement of agreement with experiment on vorticity distribution within the framework of the same model [69]. On the whole the problem of adequate description of motion of turbulent rings has been advanced greatly by the above studies, but cannot yet be considered completely solved. These studies have found concrete application in developing a new turbulent-powder method of extinguishing fires in gas and petroleum wells [70].

Lavrent'ev's idea of reducing the resistance to motion of a body in a viscous liquid by organizing the flow in analogy to the flow in a turbulent ring was evaluated in [71]. In experiments on flow of vapor over rotating cylinders Sennitskii [72] demonstrated that because of rotation the power required for translational motion can be reduced by an amount of the order of 30%.

<u>Linear (tornado-like) Vortices</u>. At the present time the formation mechanisms and structures of intense atmospheric vortices (hurricanes, waterspouts, tornados, "dust devils") have not been fully clarified. The difficulties of studying these phenomena under natural conditions has stimulated creation of laboratory models of tornado-like vortices. However

it has not been possible to reproduce the entire set of natural conditions accompanying atmospheric vortices on a small scale. Yet laboratory modeling, observation, and study of tornado-like vortices under fully controlled conditions has permitted a deeper penetration into the physics of such flows and encouraged the development of useful new ideas for explanation of phenomena in the atmosphere.

A review of recent studies in this field was presented in [73]. From experimental determination of the structure and parameters of a tornado-like vortex formed in a rotating liquid heated from below Nikulin [74] constructed an approximate model and obtained the relationship

$$V^2 \cong 10\alpha\beta gh, \tag{12}$$

where V is the maximum rotational velocity, α is the specific thermal expansion coefficient, β is the temperature difference between the core of the vortex and the external flow region, g is the acceleration of gravity, and h is the height of the vortex. Equation (12) agrees well with laboratory experiments and available data on atmospheric vortices.

Flows in vortex chambers have a structure analogous to that of tornado-like vortices. Studies of this type of turbulent flow were described by Gol'dshtik [75]. There, and also in [76] a great deal of experimental material on the structure and basic parameters of such flows was presented. The principle of a minimum in the rate of dissipation of kinetic energy into thermal was proposed for calculation of turbulent flows, making it possible to determine turbulent viscosity without use of empirical constants. For the case of a vortex chamber the turbulent viscosity is a function of flow torsion and the energy dissipation rate has two minima of differing depth [77]. These correspond to two different flow regimes — direct flow and circulation (with an internal circulation zone — a descending flow in the tornado core). Fow low torsion values only the direct flow regime is possible (there is only one minimum). With increasing torsion the second minimum appears, corresponding to the circulation flow, which with further increase in torsion becomes deeper than the first, and the circulation regime is realized. For a given torsion two regimes can be achieved. Experiments have observed a change in regime with slight change of the flow input parameters. The theoretical results are in good agreement with experimental data.

Gol'dshtik [75] solved the steady state self-similar problem of interaction of a turbulent filament with a plane in precise formulation. It was shown that the problem is uniquely soluble at small Reynolds numbers and has no solution at Reynolds numbers exceeding some critical value. The physical meaning of this result still remains unclear.

Hydrodynamic Stability. Two directions have developed intensely within this branch of hydrodynamics: stability of laminar flows of viscous liquids, in connection with the problem of laminar-turbulent transition, and stability of rotating flows of an ideal liquid, in connection with the tornado problem.

In [78, 79] experimental measurements were made of the correlation function spectrum for one velocity component in a Couette flow with rotating external cylinder. The observed evolution of the spectrum reflects a sequential change in flow regimes (bifurcation) and offers convincing evidence in favor of modern theoretical concepts of the mechanism of the transition to turbulence — the development of a singular attractor in the phase space of some (adequate) finitely dimensioned dynamic system. This conclusion has been confirmed by numerical experiment on a model.

The experiments of [80, 81] studies self-oscillation regimes, development of three-dimensional structures, and resonant interactions in Poiseuille flow and in a boundary layer in the laminar-turbulent transition region. Characteristic turbulent "lambda structures" were observed, appearing upon development of three-dimensional self-oscillations.

A monograph by Gol'dshtik and Shtern [82] was dedicated to solution of flow stability problems in channels and boundary layers, and jet and MHD flows. Effective numerical methods were developed, permitting reliable calculation of eigenvalues for any flow parameter value and determination of the stability model. The escalator model was proposed for generation of finite amplitude self-oscillations, and the character of branching of such solutions was studied. Based on this approach, [83] performed a calculation of threshold self-oscillations in a Poiseuille flow. First, a periodic wave branches from the initial flow, which can be modeled satisfactorily by the monoharmonic approximation. The following bifurcation develops by a parametric resonance mechanism upon interaction of the fundamental wave and a symmetric pair of oblique waves with doubled (as compared to the fundamental) period. The self-oscillations which develop are three-dimensional, and their structure corresponds to the experimentally observed "lambda structures" [80, 81]. A threshold curve was obtained describing the dependence of critical Reynolds number on perturbation level, which agreed with experimental data on the laminar-turbulent transition.

Vladimirov's studies of stability of rotating flow of an ideal incompressible liquid [14, 84] made use of an analogy between the effects of density stratification and rotation. It was shown in linear formulation that any rotating flow, with the exception of solid body rotation, is unstable: there exist perturbations (belonging to the continuous spectrum) which increase by a power law with time. Sufficient stability conditions were found for perturbations of the discrete spectrum, distinguishing a wide class of flows. In the nonlinear problem classes of flows were distinguished, for which the analogy between density stratification and rotation transforms to an equivalence (or closeness) of the exact equationstions of motion. In these classes, with the presence of some form of symmetry (screw, rotational, axial, translational) a number of nonlinear stability criteria were obtained [85].

A theory of resonant instability of linear vortices arising upon core deformation was proposed in [86]. Conditions for development of bending of vortices observed in a specially formulated experiment agreed well with calculation results.

Turbulence. An adequate mathematical model of the phenomenon of turbulence still remains a provocative problem. Previously proposed concepts of coherent structures and singular attractors have led to a deeper understanding of its complexity. But the absence of an effective closed mathematical description of turbulent motions greatly limits the possibilities for theoretical (and numerical) analysis. The state of affairs is such that basic progress can only be achieved in close collaboration with experiment.

At the Siberian Branch many experiments have been performed to study turbulent flows, which have produced a significant contribution to development of measurement methods and led to a number of important conclusions. A large cycle of studies in this field was summed up by Kutateladze et al. [87]. In particular, Khabakhpasheva [88] developed a method for stroboscopic flow visualization, which permitted photography of discontinuous tracks of small light-scattering particles with subsequent semiautomated computer processing of the photographs to obtain the instant velocity field, mean values, and a number of statistical characteristics of the flow. This made it possible to clarify turbulent flow structure in the direct vicinity of a wall in the viscous sublayer, where measurements with total pressure head tubes and thermoanemometers are very inaccurate. The advantages of the new method manifest themselves especially clearly in the study of flows with polymer solutions, which in some cases significantly reduce hydraulic resistance for water flow in tubes [87]. The large volume of experimental results obtained by this method provide a reliable basis for verification of theoretical models of near-wall turbulence. The ability to simultaneously measure velocity at many points in space is important for experimental detection of coherent structures at high Reynolds numbers. A similar method for measurement of free turbulent shear flow was used in [89]. There expulsion of turbulent rings from the chaotic flow region was observed.

Bukhreevs' work [90] on measurement of statistical characteristics of pressure pulsations in a hydraulic spring is of interest from the methodological viewpoint. This was one of the first studies which used a computer for processing of results, and solved a number of problems involving automation. This study was the first to establish experimentally that the probability characteristics of turbulent pulsations differ significantly from a Gaussian distribution.

In experimental studies by Bukreev, Vasil'ev, and Lytkin [91] of the turbulent wake behind bodies of various forms, evidence was obtained for the first time that despite previously held concepts, the statistical characteristics of turbulence in the wake preserve a "memory" of the details of the form of the body flowed over at very large distances down the flow. Because of this, doubts were raised as to the possibility of obtaining a universal model for description of turbulent flows in terms of single point moments. The data also indicated the significant effect of large vortices (coherent structures) on statistical flow characteristics. However the experiments in question, like other experimental studies in this field, were performed at limited Reynolds numbers, while experimental evidence exists that at sufficiently high Reynolds numbers universality may exist. This important matter requires further research.

This series of experiments involved turbulent flows in tubes for steady and pulsating flow rates. Bukreev et al. [92] obtained data on characteristic fluctuations of velocity pulsations (unique in the published literature). Veske and Sturov [93] measured all components of the turbulent stress tensor in a flow with torsion. Bukreev and Shakhin [94] observed the effect of inertia (constancy) of turbulent pulsations with change in the mean velocity profile for the case of pulsating flow rate.

In [95] an original experimental device was developed which permitted measurement of the statistical characteristics of a turbulent flow in which there is no shift in average velocity but a gradient exists in velocity pulsation intensity in the direction perpendicular to the flow. The results obtained (especially those of the last experiment) provide a rich source for verification of various theoretical models pretending to univerality.

Turbulence suppression by rotation was studied in the experiments of Tarasov and Vladimirov [96, 97] on impurity transport by a turbulent ring. It was shown that even for developed turbulence in an atmospheric vortex, turbulent transport of an impurity is practically absent. Turbulence suppression was demonstrated especially clearly in a special experiment which allowed observation of this phenomenon under steady state conditions [98], with an experiment with dye [69] being especially demonstrative. Turbulence suppression can be explained qualitatively by gyroscopic elasticity of the liquid rotating as a solid body. In the cores of concentrated vortices, where the liquid rotates almost like a solid body, turbulent velocity pulsations are of a wave character (inertial waves) and do not transport impurities [97]. The same conclusion follows from the analogy between behavior of small perturbations in a rotating and a density-stratified liquid [98]. Semiempirical models of suppression based on these studies were used to calculate the structure of a turbulent vortex ring [69].

At the present time theoretical description and calculation of turbulent flows is essentially based upon semiempirical models, the construction of which relies on analysis of experimental results and involves use of hypotheses and principles which are not well grounded. Results obtained in this manner have limited scientific significance, although this remains at present the only path for theoretical study of concrete turbulent flows. This field is widely represented in studies performed at the Siberian Branch. A large cycle of studies of the basic principles of average flow in a near-wall turbulent boundary layer without use of empirical coefficients was described by Kutateladze [99].

The third level closure model was used for numerical calculation of complex turbulent flows in [100-102] with consideration of temperature and density stratification. Results of calculation of a turbulent wake behind a cylinder and turbulent convection in a liquid layer perturbed by a fluctuating buoyancy force (the "thermocline" problem) agree well with experimental data. This model describes correctly and with good accuracy the experimentally observed phenomena of contragradient diffusion of an impurity, temperature, velocity pulsation intensity, etc., which are not described by models with lower level closure.

A model describing development of the layered structure in a continuously stratified liquid was proposed by Lyapidevskii [14]. It was shown that the mechanism of vertical mass (momentum) transport through a stable boundary is related to excitation of short waves by long ones with subsequent upsetting, leading to turbulent mixing. In [103, 104] a new model of turbulent mixing was proposed for flows with shear, considering the presence in the flow of "large vortices" and intermittency, and an analytical solution was found for the problem of decay of a tangential discontinuity.

A statistical description of weakly nonlinear interacting waves on the surface of a heavy liquid was carried out Zakharov and Filonenko [105]. A homogeneous, isotropic, weak (wave) turbulence was considered. In this case two basic assumptions are made to achieve closure: randomness of the phase distribution of individual modes and smallness of the sixth order semiinvariants, which make it possible to obtain a kinetic equation for the second order correlation function spectrum. Assuming existence of an inertial interval (in analogy to the Kolmogorov—Obdukhov interval in conventional turbulence) dividing the source and drain in wave number space, the energy spectrum was determined.

To describe supercritical regimes Gol'dshtik [106] proposed a structural turbulence model, based on the concept of a random field in the form of a set of ordered structures, randomly distributed in space and time. Application of this approach to the simple model of thermoconvective flow (the Lorentz attractor) showed that it described satisfactorily not only integral characteristics, but also the fine structure of the turbulent regime spectrum near the threshold at which they develop.

Aeroelasticity. Studies of the interaction of oscillating blades of a lattice with an incident flow are related to the problem of aeroelasticity of turbine blades. The achievements of the first stage of this effort were described in monographs by Gorelov, Kurzin, and Saran [107, 108].

Using the example of fluid flow through a lattice of thin profiles oscillating in an infrasonic flow, a number of unique features were discovered in the behavior of aerodynamic characteristics, primarily force and moment coefficients, which determine a qualitative difference from the corresponding characteristics of isolated profiles. Hydrodynamic interference of neighboring lattice profiles can lead to a significant reduction in the critical velocity at which flutter occurs. Compressibility of the fluid has a significant effect on aerodynamic characteristics of the lattice not only at large infrasonic velocities, as occurs in a steady state flow, but also at low velocities if the oscillation frequencies are high. The second unique feature is related to development of acoustical resonance, where the frequencies of the perturbing forces coincide with natural oscillation frequencies of the fluid in the lattice region. To determine acoustical resonance regimes the problem of natural oscillations of a gas flowing over a lattice of plates was solved.

The next stage of development in this field involved complication of lattice and flow models, multifaceted verification of these models, and practical application of the results obtained. Flow of an ideal incompressible fluid through a lattice of vibrating profiles of arbitrary form was considered [109]. By numerical solution of the initial-boundary problem, which considered evolution of turbulent wakes leaving the profiles, Ryabenko [110] established that for small profile oscillations the form of the waves does not have a significant effect on aerodynamic characteristics of the lattice. Therefore lattice aerodynamic characteristics obtained by solution of the linear problem can be used with a sufficient degree of reliability. Saren studied solubility conditions and found a general solution of the corresponding boundary problem. Algorithms were constructed for solution of the problem of interaction of mutually moving lattices of arbitrary profiles with consideration of turbulent wakes by Yudin [111], while problems of spatial flow of an ideal incompressible fluid through a ring lattice of thin arbitrarily shaped blades was studied by Ryabchenko [112].

The relationship between behavior of lattice aerodynamic characteristics in the vicinity of acoustic resonance regimes and the form of the corresponding natural oscillations was established by Kurzin [113]. A model was proposed in which oscillation energy loss near the lattice of plates due to radiation and formation of turbulent wakes is compensated by exponential growth of oscillation amplitude with removal from the lattice. This led to a generalization of the classical Sommerfeld radiation condition (in the absence of flow, the lattice extends along the y-axis) in the form

$$\varphi(x, y, \lambda) \cong \sum_{n=-\infty}^{+\infty} a_n \exp i\left(ny + |x|\sqrt{\lambda^2 - n^2}\right) \quad (|x| \to \infty).$$

The problem of natural oscillation values with such a radiation condition was solved by Sukhinin [114], who proved the discrete nature of the spectrum.

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HYDRODYNAMICS OF EXPLOSIONS

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The hydrodynamics of explosions, as a significant scientific specialization of the physics and mechanics of explosion processes, encompasses many problems, ranging from the generation and propagation of shock waves to the behavior of media under explosive loads. Their solution also involves the development of new experimental methods, and the creation of mathematical models of the observed effects. The last problem, however, is in many ways simplified, since the wide spectrum of flows arising in this case is described by a quite limited number of models. One of the most widely employed and simplest models is the model of an ideal incompressible liquid. It is successfully employed for the theoretical analysis of many phenomena of a typically explosive character and is based on the real possibilities of neglecting the strength and plastic properties of the media, friction forces, and compressibility under the extremely high pressures generated by the explosive loads. The use of such extremely simplified models often makes it possible to understand the essence of the process, though in making comparisons with experimental data they must also be modified.

This review is concerned with the analysis of the basic results of experimental and theoretical research on the mechanics of explosives, carried out in the Siberian Branch of the USSR Academy of Sciences over a period of 30 years from 1957 to 1986, in three important areas: shock waves in underwater explosions and cavitation, the problems of cumulation and jet flows, and explosive processes in soils.

Many of the studies enumerated below appeared owing to the attention and often the ideas of M. A. Lavrent'ev, which ultimately turned out to be the foundation for the understanding of the phenomena as a whole.

<u>Shock Waves in Underwater Explosions and Cavitation.</u> <u>Cavity Dynamics.</u> One of the most important problems in the study of underwater explosions is the analysis of the dynamics of a cavity with detonation products as a source responsible for the formation and the parameters of explosion-generated shock waves (SW). This problem is also of interest for a wide range of problems of interaction of SW with isolated cavities and an ensemble of cavities, development of bubble cavitation, formation of SW in underwater explosions of charges with a complex shape, etc. These questions were studied in detail at the Institute of Hydrodynamics of the Siberian Branch of the USSR Academy of Sciences from 1960 to 1980 and were associated with the clarification of the fundamental aspects of the effect of the compressibility of a liquid, the symmetry of flow, and the state of the gas in a pulsating cavity.

V. K. Kedrinskii [1-3] was the first to derive in the acoustic approximation a general equation describing the dynamics of a cavity in two-dimensional, cylindrical, and spherical geometries ($\nu = 0, 1, 2$). The result is based on the analysis of a one-dimensional, potential, isentropic flow of liquid, described by the system of equations (acoustic approximation)

$$c_0^{-2}\Phi_{tt} - \Phi_{rr} - \nu\Phi(1 - \nu/2)/2r^2 = 0,\quad \Phi_t = r^{\nu/2}\Omega, \qquad (1)$$

where $\Phi = r^{\nu/2}\varphi$; $\Omega = \omega + u^2/2$; $\omega = \int dp/\rho$. From here it follows that in the two-dimensional and spherical cases the system makes it possible to derive exactly and in the case of cylindrical

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